A RESULRT ON INDEXED ABSOLUTE MATRIX SUMMABILITY OF AN INFINITE SERIES

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ABSTRACT

In the present Article we established a result on "INDEXED ABSOLUTE MATRIX SUMMABILITY OF AN INFINITE SERIES" By generalizing the Theorem to $\varphi - |A, p_n; \delta|_k$, $k \ge 1$, $\delta \ge 0$ summability dealing with summability factors of infinite series, Bor. H.

Keywords: Indexed Absolute matrix Summability, Summability Factor, Infinite Series, Almost increasing sequence, Hölder inequality, Minkowski inequality.

1. INTRODUCTION

Let $\sum a_n$ be an infinite series and $\{s_n\}$ be its sequence of partial sums. Let $\{p_n\}$ be a sequence of nonnegative numbers with $P_n = \sum_{\nu=0}^n p_{\nu} \to \infty$, as $n \to \infty$ and $P_{-i} = p_{-i} = 0$, $i \ge 1$. The sequence to sequence transformation

(1.1)
$$t_n = \frac{1}{P_n} \sum_{\nu=0}^n p_{\nu} s_{\nu}$$

defines the (\overline{N}, p_n) -mean of the sequence $\{s_n\}$ generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be summable $|\overline{N}, p_n|_k, k \ge 1$, if

(1.2)
$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \left|t_n - t_{n-1}\right|^k < \infty.$$

For a lower triangular matrix $A = (a_{nk})$, we define the matrices $\overline{A} = (\overline{a}_{nk})$ and $\hat{A} = (\hat{a}_{nk})$ as follows:

(1.3)
$$\overline{a}_{nk} = \sum_{\nu=k}^{n} a_{n\nu}$$
 and $\hat{a}_{nk} = \overline{a}_{nk} - \overline{a}_{n-1,k}, \ \hat{a}_{00} = \overline{a}_{00} = a_{00}, \ n = 1, 2, \dots$

Clearly \overline{A} and \hat{A} are lower semi-matrices .Let

(1.4)
$$A_n(s) = \sum_{k=0}^n a_{nk} s_k = \sum_{k=0}^n \overline{a}_{nk} a_k$$

Then we have, $\Delta A_n(s) = \sum_{n=0}^n \hat{a}_{nk} a_k$, where $\Delta A_n(s) = A_n(s) - A_{n-1}(s)$.

Clearly $A_n(s)$ defines a sequence to sequence transformation of $s = \{s_n\}$ to $As = \{A_n(s)\}$. The series $\sum a_n$ is said to be summable $|A, p_n|_k, k \ge 1$ if