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ORTHOGONAL SPLINE COLLOCATION TECHNIQUESFOR DIFFUSION EQUATIONS

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ABSTRACT
In this paper, we use an orthogonal spline collocation method (OSCM) forthe fourth-order linear and nonlinear boundary value problem. Cubicmonomial basis functions and PiecewiseHermite cubic basis functions are used to approximate the solution for both linear and nonlinear boundary value problem. Finally, we perform several numerical experiments and usinggrid refinement analysis, we compute the order of convergence of the numerical method. Comparative to existing methods, we show that the orthogonal spline collocation methods (OSCM) gives optimal order of convergence at the knots.

Keywords: Orthogonal cubic spline collocationmethods (OCSCM), Fourth-order linear and nonlinear boundary valueproblem, Cubic monomial basis functions, Piecewise Hermite cubic basisfunctions and Almost block diagonal (ABD) matrix.

1 INTRODUCTION

The study of partial differential equations (PDEs) is a fundamental subject area of Mathematics which links important strands of Pure Mathematics to Applied and Computational Mathematics. In fact, PDEs provide a natural mathematical description of phenomena in physical, natural and biological sciences. While PDEs and their solutions exhibit rich and complex structures, the closed form analytical solutions can be found only in a few special cases and these are mostly of limited theoretical and practical interest. Therefore, it is natural to seek techniques for approximation of solutions. The rapid advancement in digital computers has resurrected the Computational Mathematics, much of which is concerned with the construction and mathematical analysis of numerical algorithms for approximating the solutions of PDEs. As a result, there is a rapid increase in the use of Mathematical Technology in Industry and Research & Development Organizations. These organizations, in return, have thrown up interesting and challenging problems in applied and Computational Mathematics and, therefore, efficient and reliable algorithms with their computational analysis have become essential for providing innovative solutions and smart strategies.

2 SPACES OF PIECEWISE POLYNOMIAL FUNCTIONS

The choice of the subspace S_h is a key element in the success of the Orthogonal Spline Collocation Methods. It is essential that S_h be chosen so that the collocation approximation can be computed efficiently and possess good approximation properties. The subspace S_h is usually chosen to be a space of piecewise polynomial functions. To define such spaces, let $\mathbf{P}_r$ denote the set of polynomials of degree $\leq \mathbf{r}$, let

$$\pi : 0 = x_0 < x_1 < x_2 < \cdots < x_N < x_{N+1} = 1$$

denote a partition of $\mathbf{I}$, and set

$$I_j = [x_{j-1}, x_j], \quad j = 1, \dots, N+1,$$

$h_j = x_j - x_{j-1}$ and $h = \max_j h_j$. We define

$$M_k^r(\pi) = \left\{ v \mid v \in C^k(I), v|_{I_j} \in \mathbf{P}_r, \quad j = 1, \dots, N+1 \right\}$$

where $C^k(I)$ denotes the space of functions which are k times continuously differentiable on $I, 0 \leq k \leq r$, and $v|_{I_j}$ denote the restriction of the function v to the interval I_j. We denote by $M_k^{r,0}(\pi)$ the space

$$M_k^r(\pi) \cap \{v \mid v(0) = v(1) = 0\}.$$